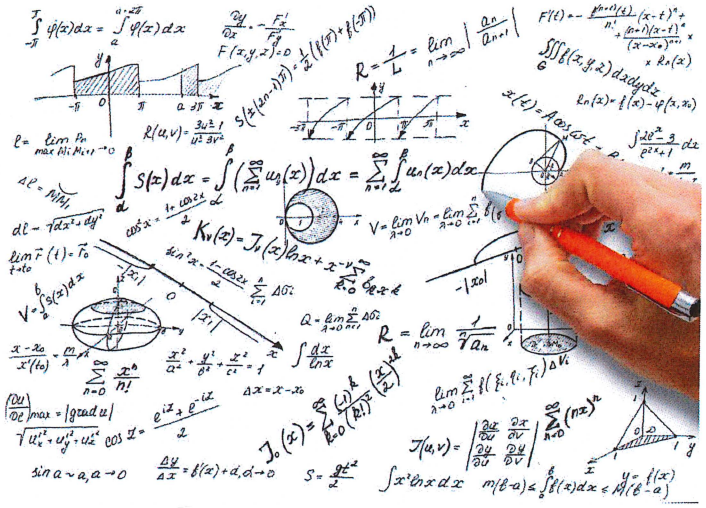
A-Level Maths Bridging Project

The Bridging Project is your first bit of *A-Level Maths Independent Study* and is a great way to brush up on some all important maths skills, keep your head in the game over summer and to prepare for working more independently when you return in September.

You can find informational videos on the following YouTube channels:

- MrBicenMaths (A-Level)
- ZeeshanZamurred (A-Level)
- HannahKettleMaths (GCSE)

Write your solutions clearly on separate sheets on paper (plain, lined or squared in fine) and bring with you **on the first day** ready to be marked in your first A-Level Maths lesson.



Alleyne's ACADEMY

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Exercise 1.1 Simple Algebraic Expressions

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make cos C the subject of the formula $c^2 = a^2 + b^2 - 2ab \cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x + 2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

Exercise 1.2 Algebraic Fractions

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7}\times 5$ (b) $\frac{5}{12}\times 3$

(c) $\frac{8}{11}\div 4$ (d) $\frac{8}{11}\div 3$

(e) $\frac{3x}{y}\times x$ (f) $\frac{3x}{y^2}\times y$

(g) $\frac{5x^3}{2y}\times 3x$ (h) $\frac{3y^4}{4x^2z}\times 2x$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u + v) = tx$ (d) $W = \frac{2}{3}\pi x^2 h$

3 Simplify the following compound fractions.

(a) $\dfrac{\frac{1}{x}+1}{\frac{1}{x}+3}$ (b) $\dfrac{\frac{2}{x}+1}{\frac{3}{x}-1}$ (c) $\dfrac{\frac{1}{x+1}+2}{\frac{1}{x+1}+1}$

4 Write as single fractions.

(a) $\frac{2}{x-1}+\frac{1}{x+3}$ (b) $\frac{2}{x-3}-\frac{1}{x+2}$ (c) $\frac{1}{2x-1}-\frac{1}{3x+2}$ (d) $\frac{3}{x+2}+1$ (e) $2-\frac{1}{x-1}$ (f) $\frac{2x}{x+1}-3$ (g) $\frac{3}{4(2x-1)}-\frac{1}{4x^2-1}$

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}}+\sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}}+\sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}}+\sqrt[3]{(x-2)^2}$

Exercise 1.3 Quadratic Expressions

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(x - 4)^2$ (c) $(2x + 1)^2$

(d) $(3x - 2)^2$ (e) $(x + 2)(x - 2)$ (f) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(x + 5)^2 + (y + 2)^2 = (x - 5)^2 + (y - 2)^2$

(c) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2+4}$ (b) $\sqrt{x^2-4x+4}$ (c) $\sqrt{x^2-1}$

(d) $\sqrt{x^2+9x}$ (e) $\sqrt{x^2-y^2}$ (f) $\sqrt{x^2+2xy+y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$ (b) $9x^2 - 12x - 1$ (c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$ (b) $4x^2 - 36$ (c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$ (e) $3x^2 - 5x + 2$ (f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x+\frac{1}{x}\right)^2$ (b) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ (c) $\left(x+\frac{2}{x}\right)\left(x-\frac{3}{x}\right)$

Exercise 1.4 Cancelling

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4+6x}{2}$ (d) $\dfrac{4\times 6x}{2}$

(e) $\dfrac{3x\times xy}{x}$ (f) $\dfrac{4x+10y}{8x+6y}$

(g) $\dfrac{4x+9y}{2x+3y}$ (h) $\dfrac{4x+6y}{6x+9y}$

(i) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ (j) $\dfrac{x-3}{3-x}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b}=\dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b}=\dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab}\div\frac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb}\div\frac{4}{3}\pi hr^2$

4 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$ (b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

(e) $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}$

Exercise 1.5 Simultaneous Equations

Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$3x + y = 10$

2 $x^2 - 4x + y^2 = 21$

$y = 3x - 21$

3 $x^2 + xy + y^2 = 1$

$x + 2y = -1$

4 $x^2 - 2xy + y^2 = 1$

$y = 2x$

Exercise 1.6 Fractional and negative powers, surds

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Trigonometry

1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

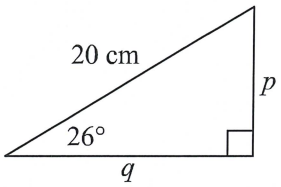
2 Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

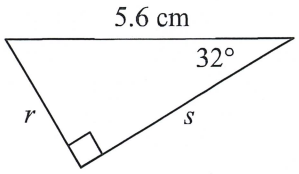
(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

3. Find expressions, of the form $a \sin \theta$ or $b \cos \theta$, for the sides labelled with letters in these triangles.

(a)

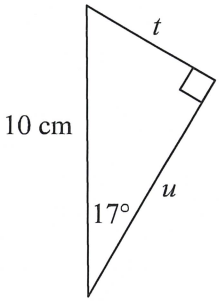


(b)

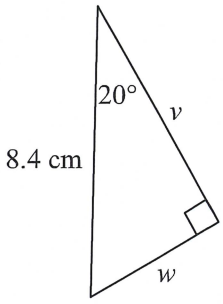


e.g. $p = 20cos26$

(c)



(d)



Exercise 3.1 Straight line graphs

1 Rearrange the following in the form $y = mx + c$. Hence find the gradient and the y-intercept of each line.

(a) $2x + y = 8$ (b) $4x - y + 9 = 0$

(c) $x + 5y = 10$ (d) $x - 3y = 15$

(e) $2x + 3y + 12 = 0$ (f) $5x - 2y = 20$

(g) $3x + 5y = 17$ (h) $7x - 4y + 18 = 0$

2 Sketch the following lines. Show on your sketches the coordinates of the intercepts of each line with the x-axis and with the y-axis.

(a) $2x + y = 8$ (b) $x + 5y = 10$

(c) $2x + 3y = 12$ (d) $3x + 5y = 30$

(e) $3x - 2y = 12$ (f) $4x + 5y + 20 = 0$

Exercise 3.2 Basic shapes of curved graphs

Sketch (do not *plot*) the general shape of the graphs of the following curves. Label the y-intercept and x-intercepts of the graphs for the quadratics. For the quadratic graphs in questions 1-6 also find the minimum point by completing the square.

Axes are not required but can be included in the questions marked with an asterix.

1 $y = x^2 - 3x + 2$

2 $y = -x^2 + 5x + 1$

3 $y = 1 - x^2$

4 $y = (x - 2)(x + 4)$

5 $y = (3 - x)(2 + x)$

6 $y = (1 - x)(5 - x)$

7 $y = x^3$

8 $y = -x^3$

9* $y = \frac{3}{x}$

10* $y = -\frac{2}{x}$

11 $y = (x - 2)(x - 3)(x + 1)$

12* $y = \frac{2}{x^2}$

13 Sketch on the same axes the general shape of the graphs of $y = x^2$ and $y = x^4$.

14 Sketch on the same axes the general shape of the graphs of $y = x^3$ and $y = x^5$.